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ДВУМЕРНАЯ ГИДРОДИНАМИЧЕСКАЯ МОДЕЛЬ НЕЛЕТУЧЕЙ НЕФТИ

TWO-DIMENSIONAL HYDRODYNAMIC MODEL OF NON-VOLATILE OIL

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Аннотация. Многие задачи, связанные с оценкой потенциала и оптимизацией работы нефтяных и газовых скважин, требуют учета течения многофазного потока в обсадной колонне, насосно-компрессорных трубах и кольцевом пространстве.
Успешное решение этих задач требует использования современных методов расчета характеристик многофазного потока в стволе скважины.
Для достижения оптимальной стратегии в современной теории и практике разработки нефтяных месторождений необходимо проводить прогнозные расчеты добычи нефти на многомерных, многофазных моделях пласта.

Ключевые слова: модель, многофазный поток, индекс, метод IMPES, насыщенность, фазовая проводимость, сетка-блок.

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Annotation. Many tasks related to potential assessment and performance optimization of oil and gas wells require taking into account the flow of multi-phase flow in the casing string, tubing and annulus.
Successful solution of these problems requires the use of modern methods for calculating the characteristics of a multiphase flow in a wellbore.
To achieve the best strategy in modern theory and practice of oil field development, it is necessary to perform predictive calculations of oil production on multi-dimensional, multiphase reservoir models.

Keywords: model, multiphase flow, index, IMPES method, saturation, phase conductivity, grid-block.

For the development of gas and gas-oil fields using horizontal wells, analytical calculation methods are required to determine the parameters of wells and the layers they penetrate [4]. One of the main problems here is the justification and choice of the technological mode of well operation.

The process of mathematical modeling – the study of an object using a mathematical model - can be divided into four interrelated stages [1–3]:

1. Formulating in mathematical terms the laws that describe the behavior of an object;
2. Solution of the direct problem, i.e. obtaining by studying the model of output data for further comparison with the results of observations of the modeling object;
3. Adaptation of the model based on the results of observation, solving inverse problems, i.e. identifying characteristics of the model that were left undefined;
4. Analysis of the model, its modernization as new information about the object under study is accumulated, a gradual transition to a new, more advanced model [5, 6].

However, instead of obtaining a formula, we find solutions to simpler equations in the form of tables of numerical values, each of which refers to discrete points in space and time within the region. This is called a numerical solution [7].

To see what is included in the design of such a model, we reduce the reservoir and consider it as one of many within the reservoir boundaries, each of which is adjacent to the surrounding ones. Schematically, such a layer can be represented as follows (Fig. 1).

The system of material balance equations for each block constitutes a mathematical model of the reservoir. By segmenting the reservoir into a system of small blocks, it is possible to take into account the values of the properties of each rock and, thus, take into account the heterogeneity of the reservoir and its anisotropy.

The spatial change in the parameters of fluid properties can be divided into separate blocks and zones of the entire system. To describe the operation of wells, we add the appropriate source for injection or for withdrawal, terms to the conservation equation for the block in which the well operates.

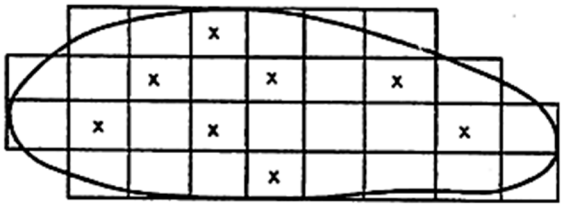

Figure 1 – Reservoir grid

The blocks labeled «x» in Figure 1 are well blocks and the material balance equations have been modified for them. Since filtration through the internal boundaries of the block is allowed, one can also trace the movements of the fluid front, monitor changes in gas/oil, water/oil contacts, and determine dynamic changes in saturation and pressure distributions.

In a numerical model, one can take two or more adjacent formations, determine rock properties, and allow flow from one block to another through adjacent faces. This constitutes a one-dimensional (1D) reservoir model. Similarly, this process can be extended to reservoirs where rock and filtration changes are allowed in more than two and three dimensions, giving rise to the 2D and 3D models [8].

It is assumed that during isothermal flow, the fluids in the reservoir are at a constant temperature and in a state of thermodynamic equilibrium. In this case, the dependences of PVT (pressure - volume - temperature) are presented as functions of the dependence of volumetric coefficients on pressure:

$$B_o = \frac{\left(V_o + V_{go}\right)_{rc}}{\left(V_o\right)_{stc}} = f\left(p_o\right) , \quad (1)$$

$$B_w = \frac{\left(V_w\right)_{rc}}{\left(V_w\right)_{stc}} = f\left(p_w\right) , \quad (2)$$

$$B_g = \frac{\left(V_g\right)_{rc}}{\left(V_g\right)_{stc}} = f\left(p_g\right) , \quad (3)$$

Mass transfer between the oil and gas phases is described by the solubility coefficient:

$$R_{so} = \left(\frac{V_{go}}{V_o}\right)_{stc} = f\left(p_o\right) , \quad (4)$$

Fluid density and pore volume are expressed as a function of pressure using an explicit or implicit equation of state.

In equations (1) – (4) the following notations are accepted:

f – Phase index (o – oil, w – water, g – gas); P_f is the pressure in the f-th phase; B_f is the volume factor of the f-th phase; V_f is the volume occupied by a fixed mass of the oil, water, gas component; V_{gf} is the volume occupied by a fixed mass of the dissolved gas component in water or oil; *stc* are normal (standard) conditions on the surface; *rc* – reservoir conditions: R_{sa} – coefficient of solubility of gas in oil; t is time.

The non-volatile oil equation is a system of equations that combines the Darcy equation of motion and the fluid mass conservation equation. The equation of motion (filtration rate) is described by Darcy's law:

$$u_f = -\frac{k_{rf}\ k}{\mu_f}\left(\nabla p_f - \gamma_f \nabla h\right), \quad (5)$$

where $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$.

The initial system of differential equations for the conservation of mass, taking into account equation (5), which describes a three-phase three-dimensional flow of fluid in a porous medium, has the form:

Oil: $$\left(\lambda_o \nabla p_o - \gamma_o \nabla h\right) = \frac{\partial}{\partial t}\left(\frac{\varphi\ s_o}{B_o}\right) + q_o , \quad (6)$$

Water: $$\left(\lambda_w \nabla p_w - \gamma_w \nabla h\right) = \frac{\partial}{\partial t}\left(\frac{\varphi\ s_w}{B_w}\right) + q_w , \quad (7)$$

Gas: $$\nabla\left[\left(R_s\lambda_o\nabla p_o-\gamma_o\nabla h\right)+\left(\lambda_g\nabla p_g-\gamma_g\nabla h\right)\right]=\frac{\partial}{\partial t}\left(\frac{\varphi R_s s_o}{B_o}+\frac{\varphi s_g}{B_g}\right)+R_s q_o+q_g\ , \quad (8)$$

where $\lambda_{f_o}=\dfrac{k\ k_{rf}}{\mu_f\ B_f}$

For a closed system (5-8), three additional relations are introduced:

$$s_o+s_w+s_g=1\ , \quad (9)$$

$$p_{cow}=p_o-p_w=f\left(s_w,s_g\right), \quad (10)$$

$$p_{cog}=p_g-p_o=f\left(s_w,s_g\right), \quad (11)$$

Notations in (5–11):

f – Phase index (*o* – oil, *w* – water, *g* – gas); *k* is the absolute permeability tensor of the porous medium; k_{rf} is the relative permeability for the *f*-th phase; λ_f is fluid mobility; B_f is the volume factor; γ_f is the specific gravity; *h* – depth relative to sea level, counting down the vertical; q_f is the flow rate of the *f*-th phase, produced from a unit reservoir volume per unit time; S_f – reservoir saturation with the *f*-th phase; R_{so} is the coefficient of gas solubility in oil (water); p_f is the pressure in the *f*-th phase; P_{cgo} is the capillary pressure in the oil-gas system; P_{cow} – capillary pressure in the oil-water system; U_f is the flow rate of the *f*-th phase; Φ – porosity; *t* is time.

For a complete formulation of the mathematical model, the original system of equations (5–11) must be supplemented with equations of state (PVT data).

The main solution methods for the multiphase flow equation are Simultaneous Solution (SS), Implicit Pressure – Explicit Saturation (IMPES), Sequential Solution (SEQ) [9]. These methods can be used to solve the equation describing one-dimensional, two-dimensional, three-dimensional, two-phase ($I = 2$) and three-phase ($I = 3$) flow problems.

The goal of the IMPES (Implicit Pressure-Explicit Saturation) method [10, 11, 12] is to obtain the pressure equation for each grid cell by combining all the flow equations to eliminate the unknown saturation. To do this, the capillary pressure and conductivity must be explicitly estimated (in time layer *n*) or at the old iterative level *k* we use explicit processing. Therefore, the pressure equation is written for each cell of the grid $n = 1, 2 ..., N$, and the resulting equations of the system are solved, either directly or iteratively. For pressure distribution in the oil phase, the second step in the IMPES method involves an explicit solution for the unknown saturation.

Equation (12) represents the final form for multiphase reservoir flow.

The pressure equation for the three-phase fixed oil model (12) is general because it reduces to the pressure equation for the oil/water and oil/gas flow models.

$$\begin{aligned}
&\sum_{m\in\psi_n}\left\{\left(B_o-R_s\ B_g\right)_n^{n+1}T_{o_{n,m}}^n+B_{w_n}^{n+1}\ T_{w_{n,m}}^n+B_{g_n}^{n+1}\left[T_{g_{n,m}}^n+\left(T_o\ R_s\right)_{n,m}^n\right]\right\}P_{o_m}^{n+1}\\
&-\left(\begin{array}{l}\left(B_o-R_s\ B_g\right)_n^{n+1}C_{op_n}+B_{w_n}^{n+1}\ C_{wp_n}+B_{g_n}^{n+1}\ C_{gp_n}\\ +\sum\limits_{m\in\psi_n}\left\{\left(B_o-R_s\ B_g\right)_n^{n+1}T_{o_{n,m}}^n+B_{w_n}^{n+1}\ T_{w_{n,m}}^n+B_{g_n}^{n+1}\left[T_{g_{n,m}}^n+\left(T_o\ R_s\right)_{n,m}^n\right]\right\}P_{o_m}^{n+1}\end{array}\right)\\
&=-\left[\left(B_o-R_s\ B_g\right)_n^{n+1}C_{op_n}+B_{w_n}^{n+1}\ C_{wp_n}+B_{g_n}^{n+1}\ C_{gp_n}\right]P_{o_m}^n\\
&-\left[\left(B_o-R_s\ B_g\right)_n^{n+1}q_{osc_n}^n+B_{w_n}^{n+1}\ q_{wsc_n}^n+B_{g_n}^{n+1}\ q_{gsc_n}^n\right]
\end{aligned}\ , \quad (12)$$

where Ψ_n – the system of near blocks is connected to the grid cell n in a natural order; m is a member of the set cell of the grid Ψn; B_o is the oil volume factor; B_w is the volumetric coefficient of water; B_g is the volumetric coefficient of the gas; R_s – gas solubility:
and

$$T_{IX} = \beta_c \frac{K_x A_x}{\Delta x} \frac{K_{rI}}{\mu_I B_I}, \tag{13}$$

where I – oil, water or gas; $T_{on,m}$ – phase conductivity of oil between grid-block n and m; $T_{wn,m}$ is the phase conductivity of water between the grid-block n and m; $T_{gon,m}$ is the phase conductivity of the gas between the grid-block n and m; P_o – pressure,
where:

$$C_{op} = \frac{V_b}{a_c \Delta t} \left[\frac{\phi'}{B_o^n} + \phi^{n+1} \left(\frac{1}{B_o} \right)' \right] \left(1 - S_w^n - S_g^n\right), \tag{14}$$

$$C_{wp} = \frac{V_b}{a_c \Delta t} \left[\frac{\phi'}{B_w^n} + \phi^{n+1} \left(\frac{1}{B_w} \right)' \right] S_w^n, \tag{15}$$

and

$$C_{gp} = \frac{V_b}{a_c \Delta t} \left(\begin{array}{l} \left\{ \left[\frac{\phi'}{B_o^n} + \phi^{n+1} \left(\frac{1}{B_o} \right)' \right] R_s^n + \left(\frac{\phi}{B_o} \right)^{n+1} R_s' \right\} \\ * \left(1 - S_w^n - S_g^n\right) + \left[\frac{\phi'}{B_g^n} + \phi^{n+1} \left(\frac{1}{B_g} \right)' \right] S_g^n \end{array} \right), \tag{16}$$

where C_{op} – coefficient $\Delta t\ p_o$ during oil expansion; C_{wp} – coefficient $\Delta t\ p_o$ at water expansion; C_{gp} – coefficient $\Delta t\ p_o$ during gas expansion

$$S_{w_n}^{n+1} = S_{w_n}^n + \frac{1}{C_{ww_n}} * \left\{ \sum_{m \in \Psi_n} \left[T_{w_{n,m}}^n \left(\Delta_m P_o^{n+1} \right) \right] - C_{wp_n} \left(P_{o_n}^{n+1} - P_{o_n}^n \right) + q_{wsc_n}^n \right\}, \tag{17}$$

where S_w – water saturation; C_{ww} – coefficient $\Delta t\ S_w$ for water expansion:

$$C_{ww} = \frac{V_b}{a_c \Delta t} \left(\frac{\phi}{B_w} \right)^{n+1}, \tag{18}$$

where ϕ – porosity

$$S_{g_n}^{n+1} = S_{g_n}^n + \frac{1}{C_{og_n}} * \left\{ \sum_{m \in \Psi_n} \left[\begin{array}{l} \left(T_{o_{n,m}}^n \left(\Delta_m P_o^{n+1} \right) \right) - C_{op_n} \left(P_{o_n}^{n+1} - P_{o_n}^n \right) + q_{wsc_n}^n \\ - C_{ow_n} \left(S_{w_n}^{n+1} - S_{w_n}^n \right) + q_{osc_n}^n \end{array} \right] \right\}, \tag{19}$$

where S_g – gas saturation; C_{og} – coefficient $\Delta t S_g$ during oil expansion

$$C_{og} = -\frac{V_b}{a_c \Delta t} \left(\frac{\phi}{B_o} \right)^{n+1}, \tag{20}$$

Δ – difference, difference operator; γ_o – specific gravity of oil; Z – grid-block height; C_{ow} – coefficient $\Delta t S_w$ during oil expansion;

$$C_{ow} = -\frac{V_b}{a_c \Delta\ t}\left(\frac{\phi}{B_o}\right)^{n+1}, \tag{21}$$

Cwg – coefficient ΔtSg for water expansion; $C_{wg} = 0$; C_{gg} – coefficient $\Delta t S_g$ during gas expansion

$$C_{gg} = \frac{V_b}{a_c \Delta\ t}\left[\left(\frac{\phi}{B_g}\right)^{n+1} - \left(\frac{\phi}{B_o}\right)^{n+1} R_s^{n+1}\right], \tag{22}$$

where

$$\phi^{/} = \frac{\left(\phi^{n+1} - \phi^{n}\right)}{\left(P_o^{n+1} - P_o^{n}\right)}, \tag{23}$$

$$R_s^{/} = \frac{\left(R_s^{n+1} - R_s^{n}\right)}{\left(P_o^{n+1} - P_o^{n}\right)}, \tag{24}$$

and

$$\left(\frac{1}{B_l}\right)^{/} = \frac{\left[\left(1/B_l^{n+1}\right) - \left(1/B_l^{n}\right)\right]}{\left(P_o^{n+1} - P_o^{n}\right)}, \tag{25}$$

where l – oil, water or gas.

Conclusion

The considered approach essentially removes all shortcomings of the material balance equation. Because fluid properties vary from point to point within a simple balance model, this is sometimes referred to as a zero-dimensional model.

The pressure equation for the three-phase solid oil model is common because it reduces to the pressure equation for the oil/water and oil/gas flow models.

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